Design of 2-Hinged Spandrel Braced Steel Arch Double Track Railway Bridge, 400 ft. Span

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1906

DESIGN & GENERAL DETAILS

of a

TWO-HINGED SPANDREL BRACED STEEL ARCH

DOUBLE TRACK RAILWAY BRIDGE

400 foot SPAN

A THESIS presented by

Samuel Klein.
&
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to the

President & Faculty

of the

Armour Institute of Technology

for the Degree

of

Bachelor of Science in Civil Engineering

having completed the prescribed course of study in

Civil Engineering.

Chicago

1906

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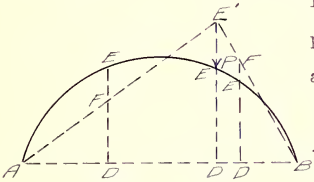
The Theory of the Spandrel Braced Arch

The usual formula for the invariability of span of an arch

$$\Sigma \frac{EF.DE}{EI} = 0$$

(Greene's Arches, Part III page 19)

is not applicable to the spandrel braced arch with horizontal top chord. In the above equation; EF is the vertical intercept between the equilibrium polygon $AE'B$ (for the load P) and the axis of the rib; *and* DE is the vertical ordinate to the equilibrium polygon, E is the co-efficient of elasticity, which is usually constant and may be taken outside of the summation sign. I is the moment of inertia of successive cross sections of the arch. In the braced arch this moment of inertia is variable and must be included in the summation; also, the values of this moment of inertia cannot be determined until the sections are known, and these two facts make it impracticable to apply this formula to the braced arch.



The omission of a center hinge renders the horizontal thrust at the abutments a statically indeterminate quantity. However, there are two practical methods of determining this thrust; 1st, by the application of the theorem of "least work", and 2d, by the elastic theory.

To illustrate how the same results are obtained by application of either of these well-known theories, both methods of obtaining the value of the horizontal thrust H will be given.

Determination of H (First Method).

Application of the Theory of Least Work.

The following equations for determining H are taken from Mueller-Breslau's, "Statik der Baukonstruction", and Du Bois', "Framed Structures".

In Fig. 1., let the span AB be equal to L, and let P be the load at a distance KL from the left end, where K is any given fraction. Let x and y be the co-ordinates of the center of moments for any member where the moment is M. Let p be the lever arm of the same member with respect to this center.

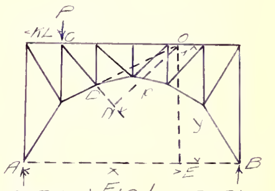


Fig. 1.

Thus, for the member CD, the center of moments is O. $AE=x$, and $EO=y$. ON, the perpendicular dropped from O upon CD produced, equals p. The stress in any member is $\frac{M}{p}$; and if we let a be the sectional area of the member, and l its length, then

the work of straining that member is

$$w = \frac{M^2 l}{2Eap^2}. \qquad (1)$$

The total work of straining all the members then is

$$\Sigma w = W = \Sigma \frac{M^2 l}{2Eap^2}. \qquad (2)$$

Now, for any member to the left of P, we have

$$M = Hy - P_1 x = Hy - P(1-k)x,$$

and for any member on the right, we have

$$M = Hy - P_1 x + P(x-kL) = Hy - P(1-k)x + P(x-kL).$$

Substituting these values of M in $Eq. 2$, we get

$$W = \Sigma_0^{kL}\left[Hy - P(1-k)x\right]^2 \frac{l}{2Eap^2} + \Sigma_{kL}^{L}\left[Hy - P(1-k)x + P(x-kL)\right]^2 \frac{l}{2Eap^2}. \qquad (3)$$

By the principle of least work, the total work W must be a minimum. Differentiating $Eq. (3)$ with respect to H, and putting the first derivative equal to zero, we get the value of H which will make the work, W, a minimum.

$$\frac{d(W)}{d(H)} = \Sigma_0^{kL}\left[Hy^2 - Py(1-k)x\right]\frac{l}{Eap^2} + \Sigma_{kL}^{L}\left[Hy^2 - P(1-k)xy + P(x-kL)y\right]\frac{l}{Eap^2} = 0;$$

hence, since E is constant,

$$H = \frac{P(1-k)\Sigma_0^{kL}\frac{xyl}{p^2a} + P\Sigma_{kL}^{L}\left[(1-k)\frac{xyl}{p^2a} - (x-kL)\frac{yl}{p^2a}\right]}{\Sigma_0^{L}\frac{y^2 l}{p^2a}}. \qquad (4)$$

Let S_0 be the stress in any member due to a unit horizontal thrust at the abutment, <u>and let S_1 be the stress due</u>

<u>to a unit load at P.</u> Then we have, for any member to the left of P,

$$S_0 p = 1 \times y, \qquad S_1 p = 1 \times (1-K) x,$$

$$\text{or} \qquad S_0 = \frac{y}{p}, \qquad S_1 = \frac{(1-K)x}{p},$$

Multiplying these two equations, we have, for any member on the left,

$$S_1 S_0 = (1-K)\frac{xy}{p^2}.$$

For any member to the right of P, we have

$$S_0 p = 1 \times y, \qquad S_1 p = 1 \times (1-K) x - 1 \times (x - KL).$$

Hence for any member to the right of P,

$$S_0 S_1 = (1-K)\frac{xy}{p^2} - (x-KL) \times \frac{y}{p^2}.$$

Substituting $S_1 S_0$ for their above values in $Eq(4)$, we get

$$H = P\frac{\sum_0^{KL}\frac{S_0 S_1 l}{a} + \sum_{KL}^{L}\frac{S_0 S_1 l}{a}}{\sum_0^L \frac{S_0^2 l}{a}},$$

$$= \frac{\sum_0^L \frac{S_0 S_1 l}{a}}{\sum_0^L \frac{S_0^2 l}{a}} P. \qquad (5)$$

The above value for H is that derived by Du Bois, and as is seen, necessitates the computation of the stresses in all the members due to a unit load at each panel point. In practice this would prove to be very tedious work. A somewhat similar formula, but better adapted to practice, may

be obtained by slight changes in the foregoing expressions, as follows:

As before, for any member to the left of P, we have

$$M_1 = Hy - P_1 x = Hy - P(1-K)x.$$

For the members to the right of P, we may consider the right abutment to be the origin of co-ordinates; or the load P may be assumed on the symmetrical panel point to the right of the center. The reaction at the left abutment on this assumption is, $P_2 = PK$

So for these members

$$M_1 = Hy \mp P_2 x = Hy - PKx.$$

By substituting these values of M_1 for M in $Eq(E)$, we get

$$W = \Sigma_0^{KL}\left[Hy - P(1-K)x\right]^2 \frac{l}{2Eap^2} + \Sigma_0^{L-KL}\left[Hy - PKx\right]^2 \frac{l}{2Eap^2},$$

$$\frac{d(W)}{d(H)} = \Sigma_0^{KL}\left[Hy^2 - P(1-K)xy\right]\frac{l}{2Eap^2} + \Sigma_0^{L-KL}\left[Hy^2 - PKxy\right]\frac{l}{2Eap^2} = 0,$$

$$\therefore\ H = \frac{P(1-K)\Sigma_0^{KL}\frac{xy^2 l}{p^2 a} + PK\Sigma_0^{L-KL}\frac{xy^2 l}{p^2 a}}{\Sigma_0^{KL}\frac{y^2 l}{p^2 a} + \Sigma_0^{L-KL}\frac{y^2 l}{p^2 a}},$$

$$= \frac{P_1\Sigma_0^{KL}\frac{xy^2 l}{p^2 a} + P_2\Sigma_0^{L-KL}\frac{xy^2 l}{p^2 a}}{\Sigma_0^{L}\frac{y^2 l}{p^2 a}}. \qquad (5)$$

As before, let S_0 be the stress in any member due to a unit thrust at the abutment; but let S_1 now be the stress in any member due to <u>a unit vertical reaction at the abutment</u>.

Then

$$S_0 p = 1 \times y, \qquad S_1 p = 1 \times x,$$

$$\text{or} \quad S_0 = \frac{y}{p}, \qquad S_1 = \frac{x}{p},$$

$$S_0 S_1 = \frac{xy}{p^2}, \qquad S_0^2 = \frac{y^2}{p^2},$$

By substituting these values in $Eq.(5)$, we get

$$H = \frac{P_1 \Sigma_0^{nL} S_0 S_1 \frac{l}{a} + P_2 \Sigma_0^{L\ nL} S_0 S_1 \frac{l}{a}}{\Sigma_0^{L} S_0^2 \frac{l}{a}} .$$

This is the value of H in a more useful form.

<u>Determination of H.</u> (Second Method)

<u>Application of the Elastic Theory.</u>

This method was first given by Prof. Clerk-Maxwell in "The Cyclopaedia Britannica" and is presented in detail by Prof. Greene in his book on Arches. Mr. R. S. Buck gives the method in his paper on the Niagara Arch.

In the arch of *Fig. 2*, let us consider the right end as fixed and the left end free to move. Now if we consider any member, as "GE", as being strained due to a stress set up in that member, and if we assume no other member being strained, then the span of the arch will be changed by an amount ΔL , <u>which will be proportional to the horizontal thrust at the free end</u>. This thrust, of course, is great enough to prevent all movement and by finding the amount of this possible movement we find the amount of the thrust which resists it.

In the case of the member "GE", the motion can be considered as taking place about the center "F", which is the center of moments for the member being considered. We may now write

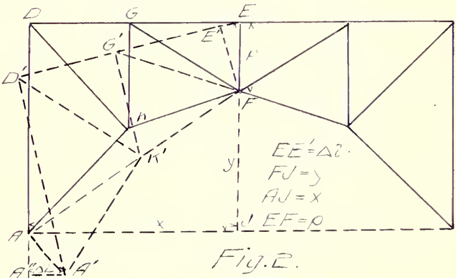


Fig. 2.

$$AA' : EE' :: AF : EF,$$

$$AA' = \frac{EE' \times AF}{EF} =$$

$$= \frac{\Delta l}{\rho} AF.$$

In similar triangles AAA and AFJ,

$$\Delta L : AA' :: Fc : AF$$

$$\therefore \quad \Delta L = \frac{AA' \times Fc}{AF} = \frac{y \times AA'}{AF}$$

$$= \frac{y}{p} \Delta l.$$

The distortion diagrams and the value of ΔL for a strain in a vertical post, in a diagonal, and in a rib member are given below:

VERTICAL POST:

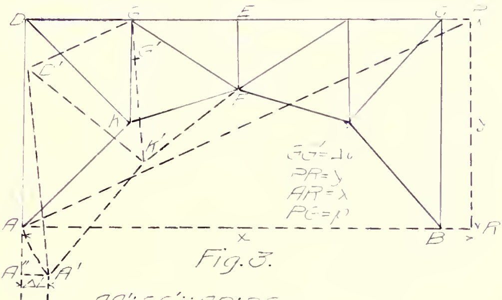


Fig. 3.

$$AA' : GG' :: AP : PG$$

$$AA' = \frac{GG' \times AP}{PG} =$$

$$= \frac{\Delta l}{p} AP$$

In Similar triangles AA'A'' and APR.

$$\Delta L : AA' :: PR : AP.$$

$$\Delta L = \frac{AA' \times PR}{AP} = \frac{y \times AA'}{AP}$$

$$= \frac{y}{p} \Delta l.$$

DIAGONAL BRACE:

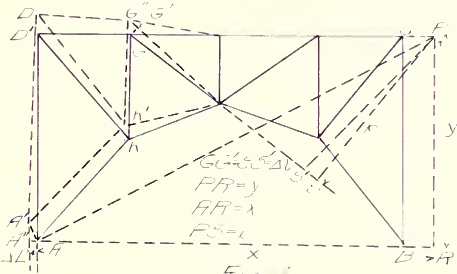


Fig. 4.

$$AA' : SS' :: AP : PS$$

$$AA' = \frac{\Delta l \times AP}{PS}$$

$$= \frac{\Delta l}{l} AP$$

In similar triangles $AA'A''$ and APR,

$$\Delta L : AA' :: PR : AP$$

$$\Delta L = \frac{AA' \times PR}{AP} = \frac{y \times AA'}{AP}$$

$$= \frac{y}{p} \Delta l.$$

RIB MEMBER:

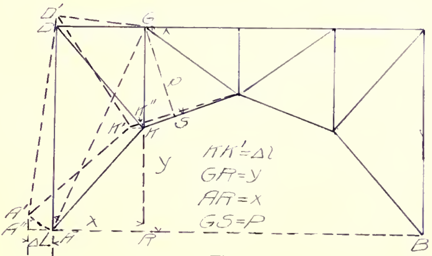


Fig. 5.

$$AA':K'K''::AG:GS$$

$$AA' = \frac{KK' \times AG}{GS} = \frac{\Delta l \times AG}{GS}$$

$$= \frac{\Delta l}{\rho} \times AG$$

In similar triangles $AA'A''$ and AGR

$$\Delta L : AA' :: GR : AG$$

$$\Delta L = \frac{AA' \times GR}{AG} = \frac{y \times AA'}{AG}$$

$$= \frac{y}{\rho} \Delta l. \qquad (7)$$

From Hook's Law

$$\Delta l = \frac{Tl}{AE}, \qquad (8)$$

where

T is the stress in a member,

E is the co-efficient of elasticity,

A is the cross-sectional area of the member,

l is the length of the member, and

Δl is the change in length, or strain due to stress, T.

Now by substituting the value of Δl given in Eq(8), for Δl in Eq(7), we get

$$\Delta L = \frac{T}{EA} \frac{ly}{\rho}. \qquad (9)$$

Any reaction at the abutments having horizontal and vertical components, H and P, will produce a stress, T, in any piece which will give, by the method of moments

$$T = \frac{Hy + Px}{\rho}. \qquad (10)$$

By substituting this value of T in Eq.(9), we get

$$\Delta l = \frac{Hy^2+Pxy}{p^2}\frac{l}{EA}.$$

The above quantity may be calculated to give the movement of the free end due to the deformation of each member of the truss. The error introduced due to the slight yielding of all the other members at the same time is inconsiderable as long as there is no sensible change in the outline of the truss.

The total change of span will then be the sum of the changes due to each member, *or*

$$\Sigma \Delta L = H\sum_0^L \frac{y^2 l}{p^2 EA} + P_1 \sum_0^{KL} \frac{xyl}{p^2 Ea} + P_2 \sum_0^{L-KL} \frac{xyl}{p^2 EA} = 0. \qquad (11)$$

Now the abutments do not yield and the span remains constant, hence ΔL must be zero.

So, from Eq.(11) we have, by solving for H,

$$H = \frac{P_1 \sum_0^{KL} \frac{xy}{p^2}\frac{l}{EA} + P_2 \sum_0^{L-KL} \frac{xy}{p^2}\frac{l}{EA}}{\sum_0^L \frac{y^2 l}{p^2 EA}}.$$

The above value for H is the same as that given in Eq.(5) and derived by the other method.

In the Engineering Record, Vol. 46, page 435, is given a simplified formula for H as derived by Mr. Gunwald Aus and approved by Mr. Theodore Cooper, Consulting Engineer. The modification is obtained by developing the formulas for h for two symmetrical panels and summarizing them. After finding the value of h for one or two symmetrical members for each loading, the value of every other h for any member can be written out mechanically, as these values have an easily established relation to each other. The summation of the values of h for any particular load gives the total H .

Mr. Aus's formulas would greatly simplify the calculations for H , but they are unfortunately not correct. Mr. Aus, as a preliminary step, derives the value of the change in span due to the change in length of any one member as

$$\Delta l = (V S_1 S_0 + h S_0^2)\frac{l}{EA}.$$

V is the vertical reaction at the abutment due to the load P on the arch and h is the horizontal thrust due to a stress in the one member considered alone. Here lies the first error; the h used in the formula must equal the total H due to stresses in all members, or, in other words, due to the load P . If it were possible to find ~~the~~ a vertical reaction due to the one member alone, then the corresponding h due to this member could be used; but V being the total vertical reaction, then h must also be the total horizontal thrust actually at the abutment and due to the stresses in <u>all</u> the members.

However, overlooking Mr. Aus's error, let us follow him farther in his derivation of H.

$$\Delta l = (V S_0 S_1 + h S_0^2)\frac{l}{EA} \qquad (1)$$

now since the abutments are fixed Δl must equal zero, and solving Eg. (/), we get

$$h = \frac{S_0 S_1 \frac{l}{EA}}{S_0^2 \frac{l}{EA}} V, \qquad (2)$$

and then the total H due to stresses in all the members is

$$\Sigma h = H = \frac{\Sigma S_0 S_1 \frac{l}{EA}}{\Sigma S_0^2 \frac{l}{EA}} V. \qquad (3)$$

At first glance, notwithstanding his error, Mr. Aus apparently derives the correct value of H.

But he is here guilty of another error in writing

$$\Sigma h = H = \frac{\Sigma S_0 S_1 \frac{l}{EA}}{\Sigma S_0^2 \frac{l}{EA}} V \qquad (3)$$

instead of

$$\Sigma h = H = \Sigma \frac{S_0 S_1 \frac{l}{EA}}{S_0^2 \frac{l}{EA}} V. \qquad (4)$$

In other words Mr. Aus boldly states that the summation of a series of fractions equals the summation of the numerators divided by the summation of the denominators.

Returning to equation (3),

$$H = \frac{\Sigma S_1 S_0 \frac{l}{EA}}{\Sigma S_0^2 \frac{l}{EA}} V$$

Mr. Aus assumes as a first approximation that the areas and lengths of all the members are equal. Hence he writes

$$H = \frac{\Sigma S_0 S_1}{\Sigma S_0^2} V.$$

Therefore

$$h = \frac{S_0 S_1}{S_0^2} V = \frac{S_1}{S_0} V$$

If Eq. (2) is correct there is no need of <u>assuming</u> the lengths and areas equal. We can write at once, by cancelling $S_0 \frac{l}{EA}$ from the fraction,

$$h = \frac{V S_1}{S_0}.$$

This is Mr. Aus's simple formula.

To sum up this criticism; Mr. Aus does not dispute the correctness of Prof. Greene's formula, and apparently derives it, but in the application of his own formulas to the design of the Rio Grande Arch he actually gets $H = \Sigma \frac{S_0 S_1 \frac{l}{EA}}{S_0^2 \frac{l}{EA}} V$,

which he states equals $\frac{\Sigma S_0 S_1 \frac{l}{EA}}{\Sigma S_0^2 \frac{l}{EA}} V$.

The Computation of Stresses in a 400 foot Railway Arch.

The arch is of the spandrel braced type with two hinges. The lower chord, or rib, is parabolic in form and the upper chord is horizontal, and the two chords are braced together by a system of vertical and diagonal web members. The span of the arch, measured between centers of the end hinges, is four hundred feet (400 feet) and is divided by the vertical posts into sixteen panels of twenty-five feet (25 feet) each. The trusses lie in planes battered one in eight (1 in 8) to the vertical. Measured in the plane of the truss, the rise of the rib is eighty feet (80 feet) and the depth of the truss at the crown is twelve feet (12 feet).

Stresses due to Live Vertical Loads.

A moving load of 6000 pounds per lineal foot of track was assumed, making the concentrations at panel points 150,000 pounds, or 150 kips.

Equation (10), on page (11), gives us the stress in any member due to a panel load W as

$$T = \frac{Hy + Px}{p} = H\frac{y}{p} + P\frac{x}{p}.$$

We can readily find the vertical reaction, P, in any case, but H is not so easily obtained. To get its value we must use the formula given on page 11;

$$H = \frac{P_1 \sum_0^{KL} \frac{xy}{p^2}\frac{l}{EA} + P_2 \sum_0^{L-KL} \frac{xy}{p^2}\frac{l}{EA}}{\sum_0^{L} \frac{y^2}{p^2}\frac{l}{EA}}.$$

It is seen from this formula that the values of the sectional areas of the members are necessary to get a correct value of H. As a first approximation these areas were assumed to be equal and the expression $\frac{1}{EA}$ dropped out of the formula. It was now necessary to get the expressions

$$\sum_0^{KL} \frac{xy}{p^2} l \;, \quad \sum_0^{L-KL} \frac{xy}{p^2} l \;, \text{ and } \quad \sum_0^{L} \frac{y^2}{p^2} l.$$

In order to get these values a table was constructed, and the work was systematized. This table is shown on PLATE I. To begin with, the values of l, p, x, y, $\frac{x}{p}$, and $\frac{y}{p}$, for each member were carefully calculated to thousandths of a foot. On PLATE II, in the upper right hand corner, is a diagram with values of $\frac{x}{p}$ and $\frac{y}{p}$ written ~~with~~ along each member. These values were obtained graphically and are the stresses due to unit vertical and horizontal reactions respectively. These values give a check on the values as

obtained analytically. It is noted that the sign of $\frac{y}{\rho}$ is plus for the upper chord and vertical members, and negative for the lower chord and diagonal members. The sign of $\frac{x}{\rho}$ is plus for lower chord members, and negative for the upper chord and vertical members. The sign of $\frac{\lambda}{\rho}$ is plus for all the diagonal members except, U_5-L_6, where it has a minus sign. The sign of $\frac{xy}{\rho}$ is minus in all cases with the exception of this diagonal, U_5-L_6. The values of $\sum \frac{xy}{\rho^2} l$ ~~and~~ are minus all through. The values of $\frac{y^2}{\rho^2} l$ and $\sum \frac{y^2}{\rho^2} l$ are plus all through.

Referring again to PLATE I, in the column headed $\frac{xy}{\rho^2} l$ are tabulated the products of the quantities $\frac{x}{\rho}$, $\frac{y}{\rho}$, and l, for each member. The next column contains the summations of these products from the left end of the truss up to each member in turn. These summations are grouped according to the type of the member, whether upper chord, lower chord, diagonal, or vertical. The next column is headed $\frac{y^2}{\rho^2} l$, and contains these quantities, and their total sums, the partial summations not being required in the formula.

The right half of PLATE I has three divisions corresponding to three determinations of the value of H, and each division has eight subdivisions, one for each panel point which may be loaded. The left hand column in these small subdivisions contains the values of $\sum_0^{nL} \frac{xy}{\rho^2} l$,

(the summation for all members which have their center of moments lying to the left of the loaded panel point) and the left hand column contains the values of $\Sigma_0^{L-nL}\frac{xy}{p^2}l$ (the summation for all members which have their center of moments lying to the right of the loaded panel point). These columns are added to give the total for all four types of members and then these totals are multiplied by P_1 and P_2, to give $P_1\Sigma_0^{nL}\frac{xy}{p^2}l$ and $P_2\Sigma_0^{L-nL}\frac{xy}{p^2}l$. The sum of these two products, divided by the value of $\Sigma_0^{L}\frac{y^2}{p^2}l$, gives the value of H.

To explain the details of the method more definitely, we will illustrate by a specific instance. In the division for the first value of H consider the small table headed, "Load on 5". The first number in the left hand column, 609.831, is the value of $\Sigma_0^{nL}\frac{xy^2}{p^2}l$ for all the upper chord members from U_0-U_1 up to and including U_4-U_5. The next number, 459.823, is the value of this summation for lower chord members from L_0-L_1 to L_5-L_6; next 493.694 is the summation for the diagonals from U_1L_2 to U_4-L_5, and 243.606 is the summation for the verticals from U_1L_1 to U_5-L_5. The total, 1806.064, is the value of $\Sigma_0^{nL}\frac{xy}{p^2}l$ ---- ------- . In the left hand column, the first number is 13,246.043. This is the value of the summation, $\Sigma_0^{L-nL}\frac{xy}{p^2}l$,

for all upper chord members which have their center of moments lying to the right of panel point 5, and includes all the members from U_5-U_6 to $U_{15}-U_{16}$ inclusive. Now since the members of the truss are symmetrically placed the summation for members measured from the right abutment toward the left is the same as the summation of members measured from the left abutment an equal distance toward the right. Therefore the summation from U_5-U_6 to $U_{15}-U_{16}$ has the same value as the summation from U_0-U_1 to $U_{10}-U_{11}$. As the summations from the left abutment are given in the table, they are always used. In this same way the summations for the other three types of members, to the right of the load, were found and the total, 26,811.837, is the value of $\sum_{t}^{L-KL} \frac{xy}{p^2} l$.

For a load W on panel point 5 we have, $P_1 = \frac{11}{16} W$, and $P_2 = \frac{5}{16} W$. $\sum_0^L \frac{y^2}{p^2} l = 12,994.898$, and the formula

$$H = \frac{P_1 \sum_0^{KL} \frac{xy}{p^2} l + P_2 \sum_0^{L-KL} \frac{xy}{p^2} l}{\sum_0^L \frac{y^2}{p^2} l},$$

becomes

$$H = \frac{\frac{11}{16} W \times 1806.964 + \frac{5}{16} W \times 26{,}811.837}{12{,}994.898},$$

which can be written

$$H = \frac{11 \times 1506.964 + 5 \times 26{,}811.857}{16 \times 12994.898} W .$$

The quantities in the numerator are different for each load but the denominator is constant and equal to 207,918.368. As W is equal to 150 kips, the value of each H in kips is very readily obtained.

PLATE III gives the values of $H\frac{y}{p}$, $P\frac{x}{p}$, and $H\frac{y}{p}+P\frac{x}{p}$ for each member for a load on each panel point. The values of $H\frac{y}{p}$ and $P\frac{x}{p}$ may be either plus or minus and their algebraic sum will be either plus or minus. By adding all the values of $H\frac{y}{p}+P\frac{x}{p}$ which have a plus sign, we get the value of the maximum tension in any member due to loads on the particular panel points which give plus values. In the same way, by adding all the stresses due to loads on panel points which give minus values, we get the maximum compressive stress in a member. These maximum stresses in each member are then combined with the stresses due to dead load, wind, and temperature, and the maximum maximorum stress in the member is thus obtained. (The method of obtaining these other stresses and making the combination will be described later).

Having the maximum stress in the members, we divided by an assumed unit stress of 10,000 pounds per square inch and

obtained an approximate area. These areas were tabulated on PLATE I in the first of the four columns headed "revised areas". In the next column, under this heading, are given the values of $\frac{xy}{\rho^2}\frac{\tau}{A}$, obtained by dividing the values of $\frac{xy}{\rho^2}\tau$ in the fourth preceding column by the value of A for that particular member. Values of $\frac{y^2}{\rho^2}\frac{\tau}{A}$ are also obtained by dividing the preceding values of $\frac{y^2}{\rho^2}\tau$ by the values of A. The same method, as is described above, is then worked through to give the separate values for H.

PLATE IV contains a table with the values for $H\frac{y}{\rho}$, ~~and~~ $P\frac{x}{\rho}$, and $H\frac{y}{\rho}+P\frac{x}{\rho}$ for these revised values of H. The values of $P\frac{x}{\rho}$ remain the same as on PLATE III, but the values of $H\frac{y}{\rho}$, and the sum $H\frac{y}{\rho}+P\frac{x}{\rho}$, are of course changed, and the maximum positive and negative stresses are altered. These new maximum values combined with the dead load, wind, and temperature stresses give new maximum maximorum values for the stresses; and these are the values which were used in the final design.

These actual areas were used to obtain a third set of values for H. The work for this determination is also shown on PLATE I.

Below is given a list of the three sets of individual and total H's with their differences:

Load on	Hs-I areas equal	Diff.	Hs-II revised areas	Diff.	Hs-III actual areas
1	.1698	-.0064	.1634	-.0006	.1628
2	.3349	-.0142	.3207	.0014	.3221
3	.4663	.0045	.4708	.0082	.4790
4	.6099	.0028	.6127	.0064	.6191
5	.7390	.0025	.7415	.0098	.7513
6	.8341	.0184	.8525	.0131	.8656
7	.9147	.0227	.9374	.0206	.9580
8	.9448	.0180	.9628	.0251	.9879
Full Load	9.0822	.0784	9.1606	.1431	9.3037

Stresses due to Dead Vertical Loads.

The dead weight of the bridge was assumed to be 10,000 pounds per lineal foot, making 5,000 pounds per foot of truss. This gives a panel load of 125,000 pounds or 125 kips, 80 per cent or 100 kips concentrated on the upper chord and 20 per cent or 25 kips on the lower chord. In getting the dead load stresses the assumption was made that the truss had been given a camber sufficient to give the lower chord its parabolic form when under the deflection due to a uniform dead load. This camber is obtained by changing the length of each member by a definite calculated amount. With this assumption made the panel loads may be considered as transferred directly to the lower chord through the posts, causing stress in the lower chord members and posts only. The lower chord stresses are readily found by the graphical method and the stresses in the posts equal 80 per cent of the panel load or 100 kips.

Stresses due to Wind Loads.

Steady Wind Loads.

The steady wind load was assumed to be nine pounds per square foot of truss area, one-half to go to each chord.

The one-half on the lower chord is divided equally between the two trusses. ~~bracing (between ribs)~~. The part of the load on the upper chord is transferred by the sway bracing to the lower chord of the wind-ward truss. These horizontal wind loads were different at each panel point due to the different depths of the truss. On PLATE II is shown the developed plan of the lower lateral system and the stresses produced by the steady wind.

The horizontal steady wind load on the upper chord is equivalent to an equal horizontal load on the lower chord plus a moment about the lower chord panel point. This moment is equal to the load on the upper chord times the vertical height above the lower chord. By dividing this product by the distance between trusses at the lower panel points a vertical force is obtained, acting down on the leeward side and up on the windward side. These vertical overturning forces cause stresses in the trusses and must be considered.

The panel points of the lower chord do not lie in the same horizontal plane and therefore the wind loads at these points will also cause overturning moments equal to that of definite equivalent vertical forces at the panel points. The method of obtaining these equivalent vertical forces is described in Merriman & Jacoby, "Higher Structures" Art. 71 page 193.

These two vertical forces acting at each panel point

were added and their values expressed in kips. Now in the table on PLATE III are given the stresses in all members due to a load of 150 kips on each panel point. If these stresses be multiplied by the ratio of the vertical forces to this load of 150 kips the stresses in the members due to these vertical forces can be obtained. PLATE V contains a table showing the stresses due to the vertical forces obtained in this manner.

Live Wind Stresses.

The live wind stresses are due to the wind on the surface of trains and are assumed to be taken by the upper lateral system. The pressure on the trains is taken as 400 pounds per lineal foot or 10 kips per panel. The upper lateral system is designed as an ordinary truss with parallel chords with loads of 10 kips at the panel points. These wind loads on the train also cause overturning moments on the truss. The amount of these horizontal wind forces (10 kips) multiplied by the vertical distance of their point of application above the lower chord this product divided by the distance between the lower chords at the panel points considered gives the value of the <u>equivalent vertical forces</u> due to the live wind. The ratio of these forces in kips to the panel load of 150 kips multiplied into the stresses due to

these loads of 150 kips gives the stresses due to the live wind overturning forces. These stresses are tabulated on PLATE V.

It is to be noted that the equivalent vertical forces due to the overturning moments of both live and dead wind loads act downward on the leeward truss and are applied on the lower chord. The stresses due to the live vertical loads of 150 kips are calculated on the assumption that the loads are applied on the upper chord and since the stresses due to the vertical forces caused by wind are obtained directly from the former stresses, there must be a correction made to allow for this difference in the point of application. The correction is made in the stresses in the posts by adding a positive stress equal to the panel load at that post algebraically. All these stresses as finally obtained are true for the leeward truss only. The stresses in the windward truss will be equal in amount but opposite in sign.

Temperature Stresses.

Variations in temperature tend to produce changes in the lengths of the members of a truss and since these changes

cannot freely take place in the case of a two-hinged arch stresses will be set up in the members. Changes in the lengths of the members would cause a change in the span of the arch and the amount of this change would depend on the length of span, the range of temperature, and the co-efficient of expansion of steel. Now since the abutments are fixed the span cannot alter, but the tendency to do so will cause a thrust or pull which we will call H_t . We can determine the value of this H_t and by the method of moments easily get at the resulting stresses in the members, which we call the temperature stresses.

Referring to page 10 we find *Equation 9*

$$\Delta L = \frac{T}{EA}\frac{lY}{r}$$

now if we let all the symbols have the same significance but consider T the stress in the member due to the temperature

we have

$$T = \frac{HY}{\rho}$$

and substituting in the former expression we get

$$\Delta L = \frac{HY^2 l}{\rho^2 EA}$$

the total change in span is

$$\Sigma\Delta L = \Sigma\frac{HY^2 l}{\rho^2 EA}$$

but $\Sigma\Delta L$ is equal to $T\varepsilon L$ where T is the change in

temperature in degrees, ε is the co-efficient of linear expansion, and L is the length of span in feet, therefore

$$\Sigma \Delta L = T\varepsilon L = H_t \Sigma \frac{y^2 l}{p^2 E A}$$

solving for H_t we get

$$H_t = \frac{T\varepsilon L}{\frac{1}{E}\Sigma \frac{y^2 l}{p^2 A}}.$$

Now for the first approximation, where the areas were all considered equal, we assumed A to equal about 196 square inches and found the value of H_t as follows:

$$H = \frac{75 \times .0000065 \times 400}{\frac{12995}{196 \times 29{,}000}} = 85.6 \text{ Kips.}$$

the value of $\Sigma \frac{y^2}{p^2} l$ is given as 12,995 on PLATE I.

The value 85.5 kips multiplied by $\frac{y}{p}$ for each member gave the stresses in the members due to temperature. In the revision of areas the value of the summation $\Sigma \frac{y^2 l}{p^2 A}$ was known as 83.492 in by substituting this value in the above expression for H_t, the other quantities being the same, we obtained 67.72 kips as the thrust or pull due to temperature, and this value gave us the stresses which were used in getting the final areas.

It is to be noted that temperature stresses are either plus or minus depending on the way the temperature varies.

A range of $75°F$ either way from $50°F$ was provided for.

Combination of Stresses.

The combining of the various stresses to give the final maximum values for which the members were designed was a rather difficult matter. The live load stresses and the dead load stresses were kept separate and the former was assumed to have twice the intensity of the latter. This assumption was introducedby adding one-half of the dead load stress to the live load stress and designing the member for live load, that is, by using unit stresses for live load.

The stresses in members due to vertical loads and temperature are the same in each truss but the stresses due to wind loads are opposite in sign in the two trusses. This latter fact made it necessary to consider the direction of the wind,and it is of course obvious that only those stresses obtained with the wind acting in the same direction can be combined.

In the lower part of the tables on PLATES III and IV are a number of rows containing the total stresses in each member due to the various causes. The first two rows marked Live Load + and Live Load — contain the maximum tension and

compression, respectively, produced by the vertical moving load. The next row, marked Dead Load, contains the maximum stress caused by the dead weight of the bridge. The two rows following are lettered Live Wind + and Live Wind — and contain the maximum tension and compression, respectively, due to the equivalent vertical forces of overturning moment of the wind on the moving trains (live wind) in the members of the leeward truss. In the row entitled Steady Wind are given the stresses in the leeward truss due to the equivalent vertical forces of overturning moment caused by wind on the truss (steady wind). The two succeeding rows marked Wind Load + and Wind Load — contain the stresses in the top and bottom chords due to the part they play in the upper and lower lateral systems. These stresses are live in the case of the upper chord. The first row marked + gives the chord stress for loads on the same panels which gave the other maximum positive live stresses. The second row marked — gives these chord stresses for loads which gave the other maximum negative live stresses. These two rows contain dead stresses in case of the lower chord members. The first row refers to the leeward truss and contains positive stresses and the second row refers to the wind-ward truss and contains negative stresses. The row labelled Temperature contains stresses which may be either plus or minus depending on the way in which the temperature varies.

The last five rows in this table are pretty well

explained by their titles. Attention is called, however, to the fact that these maximum positive and negative stresses are not absolute maximums taken by themselves, but that they will give the absolute maximum maximorum when combined.

A systematic method was used to get at the four stresses which give the largest total when combined. Eight cases were worked out for each member ~~number~~ as follows; the maximum positive live load stress, the maximum negative live load stress, the maximum positive dead load stress and the maximum negative dead load stress, for both leeward and windward trusses.

If there is a reversal of stress in a member, that is, if the member alternately takes tension and compression the member is designed for a stress equal to the larger plus eighty per cent of the smaller.

Below are given numerical examples for one upper chord member and one posts to make clear how the maximum values were obtained.

UPPER-CHORD MEMBER U_4-U_5.

Leeward Truss.

Live Load Tension		Live Load Compression	
Live Load +	+433.53	Live Load −	−805.10
Live Wind +	+ 32.60	Live Wind −	− 70.50
Wind Load +	+ 92.00	Wind Load −	+160.00
	+558.13		−715.60

Dead Load Tension		Dead Load Compression	
Steady Wind	−77.10	Steady Wind	−77.10
Temperature	+201.00	Temperature	−201.00
	+123.90		−278.10

One-half dead plus live tension $= \frac{123.90}{2} + 558.13 = 620.08$

One-half dead plus live compression $= \frac{278.10}{2} + 715.60 = 854.65$

Windward Truss.

Live Load Tension

Live Load +	+433.53
Live Wind +	− 32.60
Wind Load +	− 92.00
	+308.93

Live Load Compression

Live Load −	−805.10
Live Wind −	+ 70.50
Wind Load	−160.00
	−894.60

Dead Load Tension

Steady Wind	+77.10
Temperature	+201.00
	+278.10

Dead Load Compression

Steady Wind	− 77.10
Temperature	−201.00
	−123.90

One-half dead plus live tension $= \frac{278.10}{2} + 308.93 = 448.43$

One-half dead plus live compression $= \frac{123.90}{2} + 894.60 = 956.55$

From these figures it is readily seen that to get the greatest stress we must add eighty per cent of the greatest tension (which is 620.08) to the greatest compression (which is 956.55).

$$956.55 + .80 \times 620.08 = 1452.55$$

1452.55 kips is the maximum maximorum stress for which the member is designed.

VERTICAL MEMBER U_4L_4

Leeward Truss.

Live Load Tension

Live Load +	+84.50
Live Wind +	+ 7.10
	+ 91.60

Live Load Compression

Live Load −	− 325.92
Live Wind −	− 22.30
	− 348.22
Wind Load at Vertical	+ 12.10
	−336.12

Dead Load Tension

Dead Load +	−100.00
Steady Wind +	+9.80
Temperature	+68.20
	− 22.00

Dead Load Compression

Dead Load	−100.00
Steady Wind	+ 9.80
Temperature	− 68.20
	−158.40

One-half dead plus live tension $= -\frac{22.00}{2} + 91.60 = 80.60$

One-half dead plus live compression $= \frac{158.40}{2} + 336.12 = 415.32$

Windward Truss.

Live Load Tension

Live Load +	+84.50
Live Wind +	−7.10
	+77.40

Live Load Compression

Live Load −	−325.92
Live Wind −	+22.30
	−303.62
Wind Load at Vertical	+12.10
	−315.72

Dead Load Tension

Dead Load	−100.00
Steady Wind	−9.80
Temperature	+68.20
	−41.60

Dead Load Compression

Dead Load	−100.00
Steady Wind	−9.80
Temperature	−68.20
	−178.00

one-half dead plus live tension $= \frac{-41.60}{2} + 77.40 = +56.60$

one-half dead plus live compression $= \frac{178.00}{2} + 315.72 = -404.72$

Combining the greatest compression (415.32) in this case with 80 per cent of the greatest tension (80.60) we get the maximum maximorum in this case.

$$415.30 + .80 \times 80.60 = 479.80 \text{ kips.}$$

In the above example of the post U_4-L_4 it is noted that the stress 12.10 kips due to a live wind load at the vertical is added to get the maximum compression. The maximum compression due to live wind is obtained for a load including this panel point, therefore, this stress must be added, but the tension due to live wind is obtained for a load not including this panel point, therefore, the stress is not added in the case of the maximum tension.

The maximum stresses in lower-chord members and diagonals were obtained in the same way as they were in the case of the two types of members which have been illustrated.

Certain conventional signs have been used to indicate which stresses were used to get the various maximums. Referring to PLATES III & IV, we notice a small P, N or W placed after some of the members. The letter P indicates that the positive value of the stress was used in obtaining the maximum tension and -P means that the negative value of the stress was used in getting the maximum tension. The letter N means that the negative value of the stress was used in obtaining a maximum compression and +N indicates that the positive value of the stress was used in getting the maximum compression. A letter W after a maximum stress shows that it was obtained in the windward truss. A letter N in this case means that there was no wind. The absence of a letter means that the leeward truss is considered.

Design of a 400 Foot Railway Arch.

Main Truss Members.

In the design of the bridge, "Cooper's Specifications for Railroad Bridges" were followed with but few exceptions. All the main truss members were built up of plates and angles. In the case of the chords and verticals the angles were placed outside of the webs and one cover plate was used, the other side being laced. In the diagonals the angles were turned in and a diaphram was used. The lower-chord was spliced to make up practically the full strength of the member and the upper-chord was spliced to provide for the greatest possible tension. The gusset plates at the joints were shop riveted to the upper end of the posts and to the end away from the abutment on the lower-chord members. An allowance of twenty per cent was made for field riveting.

PLATE VI gives the composition of the members and PLATE VII shows the details.

Floor System.

The details of the floor system may be found on PLATE VIII . Cooper's "E 50" concentrated engine loading was used

in the design of the stringers and floor-beams. The stringers are 45 inches deep and rest on the top flanges of the floor beams. The floor beams are 54 inches deep and are riveted to the insides of the vertical posts.

Wind Bracing.

The systems of wind bracing are to be found on PLATE IX. The upper lateral system is of the Warren type. The members are made up of four angles laced. The lower lateral system is made up in the same way but it has additional sub-diagonals and struts which give support to the lower chord members midway between panel points. Two horizontal longitudinal struts in the plane of the truss divide the length of end post into three parts and help to support all the diagonals and verticals which they intersect. The sway bracing between posts consists of horizontal struts between points where the longitudinal struts, just mentioned, intersect the posts and of diagonal members.

The details of the end-bearings are given on PLATE X.

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May 16th 1906